2.13 Sequences (Key)

Name_____ Date _____ Period _____

Find the first 6 terms and the 100th term of the explicitly-defined sequence. Show work!

1. $u_n = \dfrac{n+1}{n}$

$2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \frac{7}{6}; \frac{101}{100}$

2. $v_n = \dfrac{4}{n+2}$

$\frac{4}{3}, 1, \frac{4}{5}, \frac{2}{3}, \frac{4}{7}, \frac{1}{2}; \frac{2}{51}$

3. $c_n = n^3 - n$

$0, 6, 24, 60, 120, 210;\ 999,900$

4. $d_n = n^2 - 5n$

$-4, -6, -6, -4, 0, 6;\ 9500$

Find the first 4 terms and the eighth term of the recursively-defined sequence. Show work!

5. $a_1 = 8$ and $a_n = a_{n-1} - 4,$ for $n \geq 2$

$8, 4, 0, -4, ;\ -20$

6. $b_1 = 2$ and $b_{k+1} = 3b_k$, for $k \geq 1$

$2, 6, 18, 54;\ 4374$

7. $c_1 = 2,\ c_2 = -1,$ and $c_{k+2} = c_k + c_{k+1},$ for $k \geq 1$

$2, -1, 1, 0;\ 3$

The sequences are arithmetic. Find a) the common difference, b) the tenth term, c) a recursive rule for the *n*th term, and d) an explicit rule for the *n*th term. Show work!

8. $6, 10, 14, 18, \ldots$

a) 4

b) 42

c) $a_1 = 6$ and $a_n = a_{n-1} + 4$ for $n \geq 2$

d) $a_n = 6 + 4(n-1) = 4n + 2$

9. $-4, 1, 6, 11, \ldots$

a) 5

b) 41

c) $a_1 = -4$ and $a_n = a_{n-1} + 5$ for $n \geq 2$

d) $a_n = -4 + 5(n-1) = 5n - 9$

10. $-5, -2, 1, 4, \ldots$

a) 3

b) 22

c) $a_1 = -5$ and $a_n = a_{n-1} + 3$ for $n \geq 2$

d) $a_n = -5 + 3(n-1) = 3n - 8$

11. $-7, 4, 15, 26, \ldots$

a) 11

b) 92

c) $a_1 = -7$ and $a_n = a_{n-1} + 11$ for $n \geq 2$

d) $a_n = -7 + 11(n-1) = 11n - 19$

The sequences are geometric. Find a) the common ratio, b) the eighth term, c) a recursive rule for the *n*th term, and d) an explicit rule for the *n*th term. Show work!

12. $2, 6, 18, 54, \ldots$

a) 3

b) 4374

c) $a_1 = 2$ and $a_n = 3a_{n-1}$ for $n \geq 2$

d) $a_n = 2 \cdot 3^{n-1}$

13. $3, 6, 12, 24, \ldots$

a) 2

b) 384

c) $a_1 = 3$ and $a_n = 2a_{n-1}$ for $n \geq 2$

d) $a_n = 3 \cdot 2^{n-1}$

14. $1, -2, 4, -8, 16, \ldots$

a) -2

b) -128

c) $a_1 = 1$ and $a_n = -2a_{n-1}$ for $n \geq 2$

d) $a_n = (-2)^{n-1}$

15. $-2, 2, -2, 2, \ldots$

a) -1

b) 2

c) $a_1 = -2$ and $a_n = -a_{n-1}$ for $n \geq 2$

d) $a_n = -2(-1)^{n-1}$ *or* $a_n = 2 \cdot (-1)^n$

16. The fourth and seventh terms of an arithmetic sequence are -8 and 4, respectively. Find the first term and a recursive rule for the *n*th term. Show work!

$a_1 = -20; a_n = a_{n-1} + 4$ for $n \geq 2$

17. The second and eighth terms of a geometric sequence are 3 and 192, respectively. Find the first term, common ratio, and an explicit rule for the *n*th term.

$a_1 = \pm\frac{3}{2}; r = \pm 2;$ and $a_n = \frac{3}{2}(\pm 2)^{n-1}$ *or* $a_n = 3(\pm 2)^{n-2}$

Review Exercises

Solve the equation algebraically. State the restrictions and identify any extraneous solutions. Show work!

18. $x + 5 = \frac{14}{x}$

$x = -7, 2; \quad x \neq 0$

19. $2 - \frac{1}{x+1} = \frac{1}{x^2 + x}$

$x = \frac{1}{2}, \quad x = -1$ is extraneous, $\quad x \neq 0, -1$

20. $3(x-2)^{\frac{3}{4}} = 24$

$x = 18; \quad x \geq 2$

21. $3\sqrt{x} + 3 = 15$

$x = 16; \quad x \geq 0$